BONDS FOR THE KOBAYASHI-MASKAWA MIXING PARAMETERS IN A MODEL WITH HIERARCHICAL SYMMETRY BREAKING

BY Z. SZADKOWSKI

Institute of Physics, University of Łódź*

(Received February 27, 1985; final version received June 11, 1985)

Relation between the weak mixing angles, the Kobayashi-Maskawa phase and the masses of pseudoscalar mesons are derived in a model with hierarchical symmetry breaking. A relation between θ_3 and θ_1 is found. Limits on θ_2 are also given. It is shown that sign ($\cos\delta$) $= -$sign ($\tan\theta_2$). Flavor nonconservation at intermediate stages of hierarchical symmetry breaking leads to CP-breaking terms in the Hamiltonian density of the Gell-Mann, Oakes and Renner (GMOR) model.

PACS numbers: 12.10.En, 11.30.Ly

1

The problem of the quark mixing and the hierarchy of the chiral symmetry breaking have been investigated by many authors [1–9] since the Cabibbo angle has been introduced into SU_3 symmetry to explain the suppression of processes with strangeness nonconservation [10]. The GIM mechanism [11], a discovery of mesons with the charm and the beauty and of a universality between quarks and leptons have extended the symmetry of a strong interaction to SU_6. At the four-quark level the relation between the mixing angle and quark masses ($\tan^2\theta_C = m_d/m_s$) is well known and agrees well with the experimental data. The mixing is connected with d and s quarks. A simultaneous mixing in (d, s) and (u, c) sectors has also been taken into account [6, 7], but due to the large mass of the c quark, the influence of the mixing in the (u, c) sector can be treated as a perturbation. At the six-quark level the quark mixing is described by three Cabibbo-like flavor mixing angles and the phase parameter responsible for CP-nonconservation [12]. The charged weak current in the $SU_6 \times SU_6$ chiral symmetry

$$J_\mu = (\bar{u}, \bar{c}, \bar{t})\gamma_\mu(1-\gamma_5)U \begin{pmatrix} d \\ s \\ b \end{pmatrix} \tag{1}$$

* Address: Instytut Fizyki, Uniwersytet Łódzki, Nowotki 149/151, 90-236 Łódź, Poland.

is described by a unitary matrix U, which can be put in 21 different forms [13], however only the standard Kobayashi-Maskawa matrix [14] will be used further.

$$U = \begin{bmatrix} c_1, & s_1c_3, & s_1s_3 \\ -s_1c_2, & c_1c_2c_3-s_2s_3e^{i\delta}, & c_1c_2s_3+s_2c_3e^{i\delta} \\ s_1s_2, & -c_1s_2c_3-c_2s_3e^{i\delta}, & -c_1s_2s_3+c_2s_3e^{i\delta} \end{bmatrix}, \tag{2}$$

where $s_i = \sin\theta_i$, $c_i = \cos\theta_i$.

The matrix (2) can be expressed as follows

$$U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_2 & s_2 \\ 0 & -s_2 & c_2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\delta} \end{bmatrix} \begin{bmatrix} c_1 & s_1 & 0 \\ -s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_3 & s_3 \\ 0 & -s_3 & c_3 \end{bmatrix}, \tag{3}$$

$$U = U_2U_\delta U_1U_3, \tag{3a}$$

and it can mix quarks either in the negative or in the positive electric charge subspace. A simultaneous mixing in both spaces was also considered [9].

From the form of the matrix (3) the following variants of the quark mixing are allowed:

$$A\colon\ U = U_2(\mathrm{s-b})U_\delta U_1(\mathrm{d-s})U_3(\mathrm{s-b}) \tag{4}$$

$$B\colon\ U = U_2(\mathrm{c-t})U_\delta U_1(\mathrm{d-s})U_3(\mathrm{s-b}) \tag{5}$$

$$C\colon\ U = U_2(\mathrm{c-t})U_\delta U_1(\mathrm{u-c})U_3(\mathrm{s-b}) \tag{6}$$

$$D\colon\ U = U_2(\mathrm{c-t})U_\delta U_1(\mathrm{u-c})U_3(\mathrm{c-t}), \tag{7}$$

where $U_k(x-y)$ denotes the mixing of x and y quarks by the matrix U_k. It is known that the Cabibbo angle cannot be explained by the mixing in the (u−c) sector only [6, 7], so the variants C and D must be rejected. Let us examine the variant B.

2

The charged weak current (1) with the matrix (2) for the variant B can be expressed as follows

$$J_\mu = RJ_\mu(0)R^{-1}, \tag{8}$$

where

$$J_\mu(0) = (\bar{\mathrm{u}}, \bar{\mathrm{c}}, \bar{\mathrm{t}})\gamma_\mu(1-\gamma_5)I\begin{pmatrix}\mathrm{d}\\ \mathrm{s}\\ \mathrm{b}\end{pmatrix}, \tag{9}$$

$$R = e^{-2i\theta_3Q^{21}}e^{-2i\theta_1Q^7}e^{-iX\delta}e^{2i\theta_2Q^{32}}, \tag{10}$$

$$X = \frac{4}{\sqrt{10}}Q^{24} - \frac{1}{\sqrt{15}}Q^{35}, \tag{11}$$

where Q^k is the 6×6 matrix representation of the k-th generator of SU_6 group. To gest the values of the angles θ_i the Gell-Mann-Oakes-Renner model will be used [15].

If the electromagnetic mass splitting of u-d quarks is neglected the Hamiltonian density breaking the chiral $SU_6\times SU_6$ symmetry is given as follows

$$H_0 = c_0 u^0 + c_8 u^8 + c_{15} u^{15} + c_{24} u^{24} + c_{35} u^{35}, \tag{12}$$

where $c_0, \ldots, c_{35}$ are the symmetry breaking parameters, u^i ($i = 0,1, \ldots, 35$) are the scalar components of the $(\bar{6},6)+(6,\bar{6})$ representation of the chiral $SU_6\times SU_6$ group.

From the GMOR model, neglecting the vacuum expectation values of operators u^k ($k = 8, 15, 24, 35$) and the spectral density ϱ^{ab} as proportional to the squared parameters of the symmetry breaking [7, 16], we get the approximate relation for masses of the pseudoscalar mesons

$$m_a^2 f_a^2 \delta^{ab} = \frac{1}{\sqrt{3}}\left(\frac{c_0}{\sqrt{3}} + c_8 d_{a8b} + c_{15} d_{a15b} + c_{24} d_{a24b} + c_{35} d_{a35b}\right)(u^0)_0, \tag{13}$$

where f_a are the decay constants, d_{aib} — symmetric constants of the SU_6 group, $(u^0)_0$ — the vacuum expectation value of the operator u^0.

From (13) we obtain

$$\pi = m_\pi^2 f_\pi^2 = \frac{1}{\sqrt{3}}\left(\frac{c_0}{\sqrt{3}} + \frac{c_8}{\sqrt{3}} + \frac{c_{15}}{\sqrt{6}} + \frac{c_{24}}{\sqrt{10}} + \frac{c_{35}}{\sqrt{15}}\right)(u^0)_0, \tag{14}$$

$$K = m_K^2 f_K^2 = \frac{1}{\sqrt{3}}\left(\frac{c_0}{\sqrt{3}} - \frac{c_8}{2\sqrt{3}} + \frac{c_{15}}{\sqrt{6}} + \frac{c_{24}}{\sqrt{10}} + \frac{c_{35}}{\sqrt{15}}\right)(u^0)_0, \tag{15}$$

$$D = m_D^2 f_D^2 = \frac{1}{\sqrt{3}}\left(\frac{c_0}{\sqrt{3}} + \frac{c_8}{2\sqrt{3}} - \frac{c_{15}}{\sqrt{6}} + \frac{c_{24}}{\sqrt{10}} + \frac{c_{35}}{\sqrt{15}}\right)(u^0)_0, \tag{16}$$

$$B = m_B^2 f_B^2 = \frac{1}{\sqrt{3}}\left(\frac{c_0}{\sqrt{3}} + \frac{c_8}{2\sqrt{3}} + \frac{c_{15}}{2\sqrt{6}} - \frac{3c_{24}}{2\sqrt{10}} + \frac{c_{35}}{\sqrt{15}}\right)(u^0)_0, \tag{17}$$

$$T = m_T^2 f_T^2 = \frac{1}{\sqrt{3}}\left(\frac{c_0}{\sqrt{3}} + \frac{c_8}{2\sqrt{3}} + \frac{c_{15}}{2\sqrt{6}} + \frac{c_{24}}{2\sqrt{10}} - \frac{2c_{35}}{\sqrt{15}}\right)(u^0)_0. \tag{18}$$

By the symmetry breaking, the massless quark x can become massive if it is mixed with the other massive y. The rotation angle is then described by the masses of pseudoscalar mesons. If the $SU_n\times SU_n$ symmetry with the exact $SU_k\times SU_k$ subsymmetry is broken to the exact $SU_{k-1}\times SU_{k-1}$ symmetry, the rotation angle is a function of masses of a pseudoscalar meson belonging to n-multiplet of the $SU_n\times SU_n$ group and the meson which has become massive [19]. We demand the quarks to become massive due to the hierarchical symmetry breaking, so the highest exact symmetry of the Hamiltonian density, which can be assumed, is $SU_4\times SU_4$ (at least one quark in the each sector must be massive). Oakes and the others [1, 17, 18], in order to get the Cabibbo angle value in the

$SU_3 \times SU_3$ or $SU_4 \times SU_4$ symmetry, have rotated the Hamiltonian density breaking the chiral symmetry in the same way as the weak charged current. In a model with hierarchical symmetry breaking such a procedure cannot be used. Let us notice that from the form (5) of the rotation operator R it follows that the quarks are mixed in the following sequence: (c-t), a phase rotation, (d-s), (s-b), so for the exact $SU_4 \times SU_4$ symmetry the massless quarks d and s would be mixed as the first (in the negative electric charge subspace) and then the generation of their masses would not be possible. The quark s would become massive in the next stage of the symmetry breaking after the mixing with the massive quark b. So, in order to get the massive both d and s quarks, they should be mixed in the inverse sequence. In the first stage of the symmetry breaking the exact $SU_4 \times SU_4$ symmetry is broken to the exact $SU_2 \times SU_2$ symmetry, in the second stage even the $SU_2 \times SU_2$ symmetry is no longer exact. The next mixing stages are connected either with the mass generation of the c quark (variant B) or with the repeated mixing of massive s and b quarks (variant A). In our procedure the Hamiltonian density breaking the chiral $SU_6 \times SU_6$ symmetry will be rotated in the inverse sequence in comparison with the rotation of the weak charged current.

$$H_{SB} = R_1 H_0 R_1^{-1}, \tag{19}$$

where

$$R_1 = e^{2i\theta_2 Q^{32}} e^{-iX\delta} e^{-2i\theta_1 Q^7} e^{-2i\theta_3 Q^{21}}. \tag{19a}$$

The exact $SU_4 \times SU_4$ symmetry implies the following relations

$$c_8 = c_{15} = 0, \tag{20}$$

$$\sqrt{5}\, c_0 + c_{35} = 0. \tag{21}$$

So, the $SU_4 \times SU_4$ invariant Hamiltonian density is given as

$$H_E = c_0(u^0 - \sqrt{5}\, u^{35}) + c_{24}(u^{24} - \sqrt{\tfrac{3}{2}}\, u^{35}), \tag{22}$$

or equivalently

$$H_E = P\bar{q}_6 q_6 - V\bar{q}_5 q_5, \tag{23}$$

where

$$P = \sqrt{12}\, c_0 + V, \quad V = \frac{5}{\sqrt{10}} c_{24}. \tag{24}$$

The symmetry-breaking Hamiltonian density

$$H_{SB} = R_1 H_E R_1^{-1} \tag{25}$$

retaining the flavor-conservation part only is given as follows

$$H_{SB} = \bar{q}_6 q_6 P c_2^2 - \bar{q}_5 q_5 V c_3^2 + \bar{q}_4 q_4 P s_2^2 - \bar{q}_3 q_3 V c_1^2 s_3^2 - \bar{q}_2 q_2 V s_1^2 s_3^2. \tag{26}$$

Let us notice that the phase transformation does not produce terms $\bar{q}_i q_i$, since the operator (11) commutes with the scalar components u^k. The flavor-conservation on each stage of the symmetry breaking has been assumed. The Hamiltonian density (26) can be written as a function of the operators u^k, so the coefficients of u^k's are given as

$$c'_0 = c_0, \tag{27}$$

$$c'_3 = \frac{V}{2} s_1^2 s_3^2, \tag{28}$$

$$c'_8 = \frac{V}{2\sqrt{3}}(2c_1^2 s_3^2 - s_1^2 s_3^2), \tag{29}$$

$$c'_{15} = \frac{-1}{2\sqrt{6}}(3Ps_2^2 + Vs_3^2), \tag{30}$$

$$c'_{24} = \frac{1}{2\sqrt{10}}(4V - 5Vs_3^2 + Ps_2^2), \tag{31}$$

$$c'_{35} = \frac{-1}{2\sqrt{15}}(5P + V - 6Ps_2^2). \tag{32}$$

Now, after the symmetry breaking, the pseudoscalar masses (14–18) will be described as functions of the coefficients c'_i (i = 0, 3, 8, 15, 24, 35) [7].

$$\pi = ZVs_1^2 s_3^2, \tag{33}$$

$$K = ZVs_3^2(1 - \tfrac{1}{2} s_1^2), \tag{34}$$

$$D = -Z(Ps_2^2 - \tfrac{1}{2} Vs_1^2 s_3^2), \tag{35}$$

$$B = ZV(1 - s_3^2(1 - \tfrac{1}{2} s_1^2)), \tag{36}$$

$$T = -Z(Pc_2^2 - \tfrac{1}{2} Vs_1^2 s_3^2), \tag{37}$$

where

$$Z = -\frac{1}{2\sqrt{3}}(u^0)_0. \tag{38}$$

The Cabibbo angle θ_1 is expressed in the same form as at four-quark level in the $SU_4 \times SU_4$ symmetry [7, 19].

$$s_1^2 = \frac{2\pi}{2K + \pi} \tag{39}$$

because

$$s_1^2 s_3^2 = \frac{\pi}{K + B}, \tag{40}$$

hence

$$s_3^2 = \frac{K+\dfrac{\pi}{2}}{K+B}. \tag{41}$$

In an agreement with our prediction the angle θ_3 connected with the mixing of s and b quarks is expressed by the parameters of the strange and beautiful mesons. The angle θ_1 however, connected with mixing d and s quarks and breaking of the $SU_2 \times SU_2$ symmetry is expressed by the masses of the pion and the kaon. The angle θ_2 connected with the mixing in the (c-t) sector is given as

$$s_2^2 = \frac{D-\dfrac{\pi}{2}}{D+T-\pi}. \tag{42}$$

Let us notice that if we do not demand the flavor-conservation on each stage of the symmetry breaking, after the rotation around the 21st axis the terms $\bar{q}_5 q_3$, $\bar{q}_3 q_5$ in the broken Hamiltonian density arise. In the second stage (the rotation around the 7th axis) there will be in H_{SB} the following terms: $\bar{q}_5 q_3$, $\bar{q}_3 q_5$, $\bar{q}_2 q_3$, $\bar{q}_3 q_2$, $\bar{q}_2 q_5$, $\bar{q}_5 q_2$. Because

$$[X, \bar{q}_3 q_5] = i\delta \bar{q}_3 q_5, \tag{43}$$

$$[X, \bar{q}_5 q_3] = -i\delta \bar{q}_5 q_3, \tag{44}$$

after the phase rotation there will arise in the H_{SB} the following terms: $\bar{q}_3 q_5 e^{i\delta}$, $\bar{q}_5 q_3 e^{-i\delta}$, ... In the variant B the matrix U_2 has mixed c and t quarks so that in the flavor-conservation part of the broken Hamiltonian density the phase factor $e^{i\delta}$ cannot appear. But if the matrix U_2 mixes s and b quarks again, due to the following relations

$$e^{-2i\theta_2 Q^{21}} \bar{q}_3 q_5 e^{2i\theta_2 Q^{21}} = \bar{q}_3 q_5 c_2^2 - \bar{q}_5 q_3 s_2^2 + \tfrac{1}{2}(\bar{q}_5 q_5 - \bar{q}_3 q_3) \sin 2\theta_2, \tag{45}$$

$$e^{-2i\theta_2 Q^{21}} \bar{q}_5 q_3 e^{2i\theta_2 Q^{21}} = \bar{q}_5 q_3 c_2^2 - \bar{q}_3 q_5 s_2^2 + \tfrac{1}{2}(\bar{q}_5 q_5 - \bar{q}_3 q_3) \sin 2\theta_2 \tag{46}$$

in the broken Hamiltonian density the terms

$$\bar{q}_5 q_5 e^{i\delta}, \quad \bar{q}_5 q_5 e^{-i\delta}, \quad \bar{q}_3 q_3 e^{i\delta}, \quad \bar{q}_3 q_3 e^{-i\delta}$$

will exist.

We assume that the symmetry is broken by the quarks mixing in the following sequence: (s-b), (d-s), a phase rotation, (s-b) and the flavor will not be conserved in the intermediate stages of the symmetry breaking, but it will be conserved in the broken symmetry taken as a whole. The assumptions given above are consistent with the variant A. Let us take it into account.

3

We assume the exact $SU_4 \times SU_4$ symmetry. The Hamiltonian density is given by Eq. (23). After symmetry breaking, the flavor conserving part of the broken Hamiltonian density H_{SB} is given as

$$H_{(\Delta F=0)} = \bar{q}_6 q_6 P - \bar{q}_5 q_5 V(\alpha - A) - \bar{q}_3 q_3 V(\beta + A) - \bar{q}_2 q_2 V\gamma, \tag{47}$$

where

$$\alpha = c_1^2 s_2^2 s_3^2 + c_2^2 c_3^2, \tag{48}$$

$$\beta = c_1^2 c_2^2 s_3^2 + s_2^2 c_3^2, \tag{49}$$

$$\gamma = s_1^2 s_3^2, \tag{50}$$

$$A = \tfrac{1}{2} \cos\theta_1 \sin 2\theta_2 \sin 2\theta_3 \cos\delta. \tag{51}$$

Since

$$\alpha + \beta + \gamma = 1 \tag{52}$$

α can be eliminated from (47).

The coefficients by the operators u^k are as follows

$$c_0' = c_0, \tag{53}$$

$$c_3' = \frac{V}{2}\gamma, \tag{54}$$

$$c_8' = \frac{V}{2\sqrt{3}}(2\beta + 2A - \gamma), \tag{55}$$

$$c_{15}' = -\frac{V}{2\sqrt{6}}(\beta + A + \gamma), \tag{56}$$

$$c_{24}' = \frac{V}{2\sqrt{10}}(4 - 5(\beta + A + \gamma)), \tag{57}$$

$$c_{35}' = -\sqrt{5}\, c_0 - \sqrt{\tfrac{3}{2}}\, c_{24}. \tag{58}$$

Let us notice that the functions β and A occur in Eqs. (55–57) as a sum $\beta + A$ only. So, only two functions can be expressed independently. Because there is no mixing in the positive electric charge subspace we shall not use relations describing mesons D and T. The following relations are obeyed

$$\pi = ZV\gamma, \tag{59}$$

$$K = ZV\left(\beta + A + \frac{\gamma}{2}\right), \tag{60}$$

$$B = ZV\left(1 - \beta - A - \frac{\gamma}{2}\right) \tag{61}$$

so we immediately obtain

$$s_1^2 s_3^2 = \frac{\pi}{K+B} \tag{62}$$

as in the variant B, but at the moment the angles θ_1 and θ_3 cannot be calculated separately. Putting the experimental value

$$\cos\theta_1 = 0.9737 \quad (\sin\theta_1 = 0.2278) \tag{63}$$

as an input [20], we get

$$\sin\theta_3 = 0.136 \quad (\theta_3 = 7.8°) \tag{64}$$

for

$$m_\pi = 0.139 \text{ GeV}, \quad f_\pi = 1, \tag{65}$$

$$m_K = 0.495 \text{ GeV}, \quad f_K = 1.28 \ [21], \tag{66}$$

$$m_B = 5.2 \text{ GeV}, \quad f_B = 0.86 \ [22]. \tag{67}$$

The angle θ_3 was calculated by Fritzsch [6] also for the following quark masses ratios:

$$m_u : m_d : m_s : m_c = 1 : 1.78 : 35.7 : 285 \tag{68}$$

and the limit for the angle θ_2

$$\theta_2 < (m_c/m_t)^{1/2} = 0.33. \tag{69}$$

For the assumptions given above Fritzsch obtained the following boundary

$$\sin\theta_3 < 0.09 \quad (\theta_3 < 5°).$$

However there is no agreement between descriptions of the quark masses ratios. The other authors [23] give smaller difference between quark masses

$$m_u : m_d : m_s : m_c = 1 : 1.1 : 6.4 : 23.6. \tag{70}$$

Thus for the ratio (70) we get the following limit for the angle θ_3

$$\sin\theta_3 < 0.163. \tag{71}$$

The value of the angle θ_3 (64) is consistent with the boundary (71). The value (64) is close to the value given by Białas [8] and consistent with results in Refs. [24, 26] as well as the experimental boundary:

$$\sin\theta_3 < 0.42 \quad [27], \tag{72}$$

$$|\sin\theta_3| = 0.28 \begin{smallmatrix} +0.21 \\ -0.28 \end{smallmatrix} \quad [20]. \tag{73}$$

Let us consider the relation between the angle θ_2 and the phase parameter δ. From (59)–(61) we obtain

$$\beta+A = \frac{K-\frac{\pi}{2}}{K+B} \tag{74}$$

or equivalently

$$\frac{K+\frac{\pi}{2}}{K+B} - \frac{\gamma}{s_1^2} = s_2^2\left(1+\gamma\left(1-\frac{2}{s_1^2}\right)\right)+A \tag{75}$$

denoting

$$\xi = \frac{\left(K+\frac{\pi}{2}\right)-\frac{\pi}{s_1^2}}{K+B}, \tag{76}$$

$$\eta = 1+\frac{\pi}{K+B}\left(1-\frac{2}{s_1^2}\right), \tag{77}$$

$$\varrho = \tfrac{1}{2}\cos\theta_1 \sin 2\theta_3 \tag{78}$$

we get

$$\cos\delta = \frac{\xi-s_2^2\eta}{\varrho\sin 2\theta_2}. \tag{79}$$

It is worth noting that if we take the constraint on the Cabibbo angle θ_1 from the four--quark level [7, 19], which is the same as given by Eq. (39), the parameter ξ (76) will be exactly equal to zero, hence we get

$$\cos\delta = -\frac{\eta}{2\varrho}\tan\theta_2. \tag{80}$$

Because $|\cos\delta| \leqslant 1$, so from (80)

$$|\theta_2| < \left|\mathrm{arc\ tan}\,\frac{2\varrho}{\eta}\right| \tag{80a}$$

and we get also a boundary on the angle θ_2

$$\sin\theta_2 < 0.265 \qquad (\theta_2 < 15.4^\circ). \tag{81}$$

The value (81) is in a good agreement with the results given by Fritzsch [6]

$$9^\circ < \theta_2 < 19^\circ, \tag{82}$$

Białas [8]

$$\sin\theta_2 = 0.23, \tag{83}$$

Shrock, Treiman, Wang [20]

$$|\sin \theta_2| < 0.25 \quad \text{for} \quad m_t = 15 \text{ GeV}, \tag{84}$$

Barger, Long, Pakvasa [24]

$$\sin \theta_2 < 0.5 \quad \text{for} \quad m_t = 30 \text{ GeV}, \tag{85}$$

and experimental limits [27].

The Eq. (79) can be written as follows

$$x^2(\eta^2+4\varrho^2\cos^2\delta)-2x(\xi\eta+2\varrho^2\cos^2\delta)+\xi^2 = 0, \tag{86}$$

where

$$x = \sin^2\theta_2. \tag{87}$$

To get a real value of the angle θ_2 the determinant of the square equation (86) cannot be negative, so

$$16\varrho^2\cos^2\delta(\xi\eta+\varrho^2\cos^2\delta-\xi^2) \geqslant 0, \tag{88}$$

hence

$$1 \geqslant \cos^2\delta \geqslant \frac{\xi(\xi-\eta)}{\varrho^2}. \tag{89}$$

For (63, 64) we get

$$\eta = 0.9647, \qquad \varrho = 0.1309. \tag{90}$$

If the parameter ξ, which can be identified with a change of the Cabibbo angle description by a transition to the higher symmetries, is slightly less than zero, the phase parameter δ will be bounded ($|\xi|$ should be nearly zero, as the Cabibbo angle description should not change strongly by a transition to higher symmetries, on the other hand the Eq. (89) gives a boundary on the parameter ξ

$$\xi > -0.0175. \tag{91}$$

From (80)

$$\operatorname{sign}(\cos\delta) = -\operatorname{sign}(\tan\theta_2) \tag{92}$$

so, for the angle θ_2 lying in the first quadrant, it follows $\frac{\pi}{2} < \delta < \pi$ and from (91) there is a lower limit for the phase δ. For an input given by the Eqs. (63, 64, 66) we get

$$\xi = 0.002, \tag{93}$$

so there is no boundary on δ, since sign $\xi = +1$. Let us notice, that a small change of the f_K can change the sign of the parameter ξ. Following Fuchs [25], in a chiral perturbation theory at the $SU_3 \times SU_3$ level

$$\frac{f_K}{f_\pi} = 1+\frac{3(m_K^2-m_\pi^2)}{64\pi^2 f_\pi^2}\ln\frac{\Lambda}{4\mu^2}+O(\varepsilon), \tag{94}$$

where μ^2 is the average meson squared mass and Λ is a cut-off parameter, which is estimated to be near $4\,m_N^2$, it implies

$$f_K/f_\pi = 1.15 \tag{95}$$

so

$$\xi = -0.00179 \tag{96}$$

and we get

$$\cos^2 \delta > 0.1. \tag{97}$$

Taking into account (92) we obtain

$$109° < \delta < 180°. \tag{98}$$

Since f_K is treated as a variable and can depend on the energy scale via Λ parameter and the symmetry breaking parameters ε, the boundary of the phase due to the Eqs. (76, 89) can be expected. For sign $(\cos \delta) = -1$ there is a lower limit of the angle θ_2 also. A variant $\cos \delta > 0$ is allowed but the angle θ_2 corresponding to this variant is too severely limited and it is not consistent with the experimental data [27].

4

In this paper we have shown that the weak mixing angles at the six-quark level can be estimated in terms of the masses of pseudoscalar mesons. The calculation of mixing angles is possible by using the hierarchical symmetry breaking leading to a quark masses generation. A number of independent mixing angles that can be calculated on the ground of the given above model is equal to a number of degrees of freedom connected with the symmetry breaking and the quarks mixing in the fixed electric charge subspace (let us notice that in the variant B after the rotation around the 21st axis and next around the 7th one, even the exact $SU_2 \times SU_2$ symmetry did not remain; however the angle θ_2 connected with the mixing in the positive electric charge subspace could be calculated).

An assumption that in the hierarchical symmetry breaking the flavor does not have to be conserved on each stage of the symmetry breaking, while it is conserved in the broken symmetry taken as a whole, has allowed the author to introduce to the broken Hamiltonian density a phase angle responsible for *CP*-nonconservation.

The experimental value of the Cabibbo angle treated as an input has allowed the author to calculate the angle θ_3 and to find the relation connecting the angle θ_2 and the phase parameter δ. Limits of trigonometric functions values imply boundaries on the angle θ_2 and the phase δ. The kaon decay constant is a sensitive parameter, which can introduce *CP*-nonconservation to the chiral perturbation theory. Boundaries for the angle θ_2 and the phase δ vs. f_K can be also found.

I would like to thank Prof. W. Tybor for inspiration.

REFERENCES

[1] R. J. Oakes, *Phys. Lett.* **29B**, 683 (1969).
[2] A. Ebrahim, *Lett. Nuovo Cimento* **19**, 225 (1977).
[3] A. Ebrahim, *Lett. Nuovo Cimento* **19**, 437 (1977).
[4] A. Ebrahim, *Lett. Nuovo Cimento* **24**, 164 (1979).
[5] A. Ebrahim, *Phys. Lett.* **69B**, 229 (1977).
[6] H. Fritzsch, *Nucl. Phys.* **B155**, 189 (1979).
[7] Z. Szadkowski, *Acta Phys. Pol.* **B14**, 23 (1983).
[8] A. Białas, *Acta Phys. Pol.* **B12**, 897 (1981).
[9] S. K. Oh, J. K. Kim, *Phys. Lett.* **102B**, 312 (1981).
[10] N. Cabibbo, *Phys. Rev. Lett.* **10**, 531 (1963).
[11] S. L. Glashow, J. Iliopoulos, L. Maiani, *Phys. Rev.* **D2**, 1285 (1970).
[12] Y. Fujimoto, *Lett. Nuovo Cimento* **29**, 283 (1980).
[13] R. Mignani, *Lett. Nuovo Cimento* **28**, 529 (1980).
[14] M. Kobayashi, K. Maskawa, *Prog. Theor. Phys.* **49**, 652 (1973).
[15] M. Gell-Mann, R. J. Oakes, B. Renner, *Phys. Rev.* **175**, 2195 (1968).
[16] V. de Alfaro, S. Fubini, G. Furlan, C. Rossetti, *Currents in Hadron Physics*, North Holland Publishing Company 1973, p. 509–517.
[17] N. Cabibbo, L. Maiani, *Phys. Lett.* **28B**, 131 (1968).
[18] R. Gatto, G. Sartori, M. Tonin, *Phys. Lett.* **28B**, 128 (1968).
[19] Z. Szadkowski, *Acta Phys. Pol.* **B13**, 247 (1982).
[20] R. E. Shrock, S. B. Treiman, L. L. Wang, *Phys. Rev. Lett.* **12**, 1589 (1979).
[21] K. P. Das, N. G. Deshpande, *Phys. Rev.* **D19**, 3387 (1979).
[22] S. Fajfer, Z. Stipcevic, *Lett. Nuovo Cimento* **29**, 207 (1980).
[23] B. Bagchi, V. P. Gautam, A. Nandy, *Phys. Rev.* **D19**, 3380 (1979).
[24] V. Barger, W. F. Long, S. Pakvasa, *Phys. Rev. Lett.* **42**, 1585 (1979).
[25] N. H. Fuchs, *Lett. Nuovo Cimento* **27**, 21 (1980).
[26] E. Ma, *Phys. Lett.* **96B**, 115 (1980).
[27] K. Kleinknecht, B. Renk, *Z. Phys. C — Particles and Fields* **16**, 7 (1983).